OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4761

Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4761
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 An egg falls from rest a distance of 75 cm to the floor.

Neglecting air resistance, at what speed does it hit the floor? **[3]**

2 Fig. 2 shows a sack of rice of weight 250 N hanging in equilibrium supported by a light rope AB. End A of the rope is attached to the sack. The rope passes over a small smooth fixed pulley.

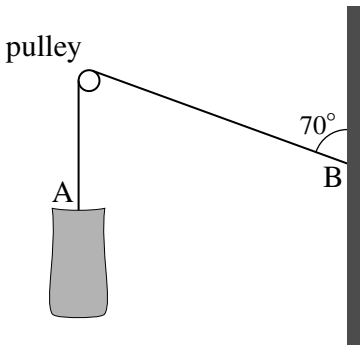


Fig. 2

Initially, end B of the rope is attached to a vertical wall as shown in Fig. 2.

(i) Calculate the horizontal and the vertical forces acting on the wall due to the rope. **[3]**

End B of the rope is now detached from the wall and attached instead to the top of the sack. The sack is in equilibrium with both sections of the rope vertical.

(ii) Calculate the tension in the rope. **[1]**

3 The three forces $\begin{pmatrix} -1 \\ 14 \\ -8 \end{pmatrix}$ N, $\begin{pmatrix} 3 \\ -9 \\ 10 \end{pmatrix}$ N and **F** N act on a body of mass 4 kg in deep space and give it an acceleration of $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ m s^{-2}.

(i) Calculate **F**. **[4]**

At one instant the velocity of the body is $\begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}$ m s^{-1}.

(ii) Calculate the velocity and also the speed of the body 3 seconds later. **[4]**

4 As shown in Fig. 4, boxes P and Q are descending vertically supported by a parachute. Box P has mass 75 kg. Box Q has mass 25 kg and hangs from box P by means of a light vertical wire. Air resistance on the boxes should be neglected.

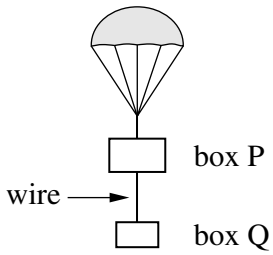


Fig. 4

At one stage the boxes are slowing in their descent with the parachute exerting an upward vertical force of 1030 N on box P. The acceleration of the boxes is a m s^{-2} upwards and the tension in the wire is T N.

(i) Draw a labelled diagram showing all the forces acting on box P and another diagram showing all the forces acting on box Q. **[2]**

(ii) Write down separate equations of motion for box P and for box Q. **[3]**

(iii) Calculate the tension in the wire. **[2]**

5 In this question the unit vectors **i** and **j** are pointing east and north respectively.

(i) Calculate the bearing of the vector $-4\mathbf{i} - 6\mathbf{j}$. **[2]**

The vector $-4\mathbf{i} - 6\mathbf{j} + k(3\mathbf{i} - 2\mathbf{j})$ is in the direction $7\mathbf{i} - 9\mathbf{j}$.

(ii) Find k. **[4]**

6 A small ball is kicked off the edge of a jetty over a calm sea. Air resistance is negligible. Fig. 6 shows

- the point of projection, O,
- the initial horizontal and vertical components of velocity,
- the point A on the jetty vertically below O and at sea level,
- the height, OA, of the jetty above the sea.

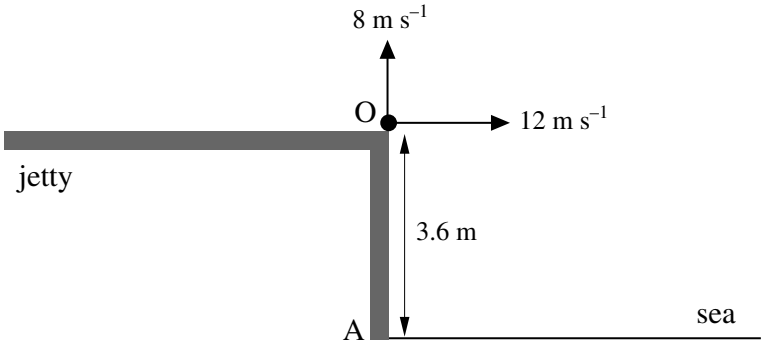


Fig. 6

The time elapsed after the ball is kicked is t seconds.

(i) Find an expression in terms of t for the height of the ball above O at time t. Find also an expression for the horizontal distance of the ball from O at this time. **[3]**

(ii) Determine how far the ball lands from A. **[5]**

Section B (36 marks)

7 A point P on a piece of machinery is moving in a vertical straight line. The displacement of P above ground level at time t seconds is y metres. The displacement-time graph for the motion during the time interval $0 \leqslant t \leqslant 4$ is shown in Fig. 7.

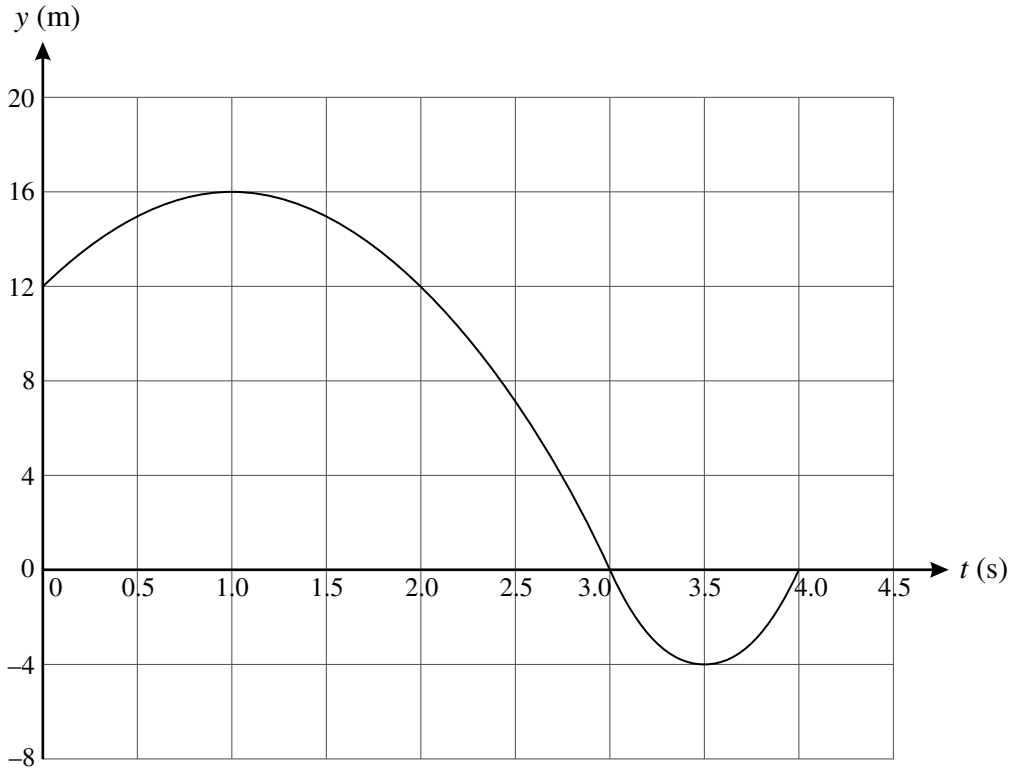


Fig. 7

(i) Using the graph, determine for the time interval $0 \leqslant t \leqslant 4$

(*A*) the greatest displacement of P above its position when $t = 0$,

(*B*) the greatest distance of P from its position when $t = 0$,

(*C*) the time interval in which P is moving downwards,

(*D*) the times when P is instantaneously at rest. **[6]**

The displacement of P in the time interval $0 \leqslant t \leqslant 3$ is given by $y = -4t^2 + 8t + 12$.

(ii) Use calculus to find expressions in terms of t for the velocity and for the acceleration of P in the interval $0 \leqslant t \leqslant 3$. **[3]**

(iii) At what times does P have a speed of $4\,\text{m s}^{-1}$ in the interval $0 \leqslant t \leqslant 3$? **[2]**

In the time interval $3 \leqslant t \leqslant 4$, P has a constant acceleration of $32\,\text{m s}^{-2}$. There is no sudden change in velocity when $t = 3$.

(iv) Find an expression in terms of t for the displacement of P in the interval $3 \leqslant t \leqslant 4$. **[5]**

8 A cylindrical tub of mass 250 kg is on a horizontal floor. Resistance to its motion other than that due to friction is negligible.

The first attempt to move the tub is by pulling it with a force of 150 N in the **i** direction, as shown in Fig. 8.1.

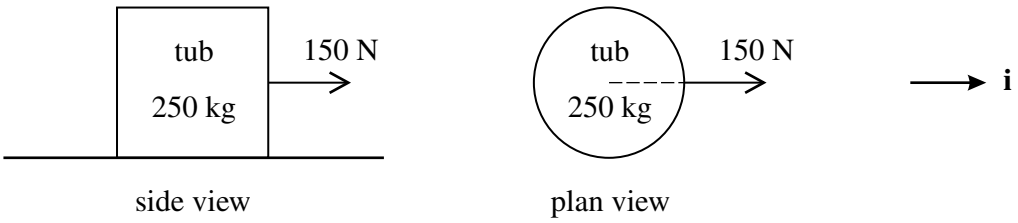


Fig. 8.1

(i) Calculate the acceleration of the tub if friction is ignored. **[2]**

In fact, there is friction and the tub does not move.

(ii) Write down the magnitude and direction of the frictional force opposing the pull. **[2]**

Two more forces are now added to the 150 N force in a second attempt to move the tub, as shown in Fig. 8.2.

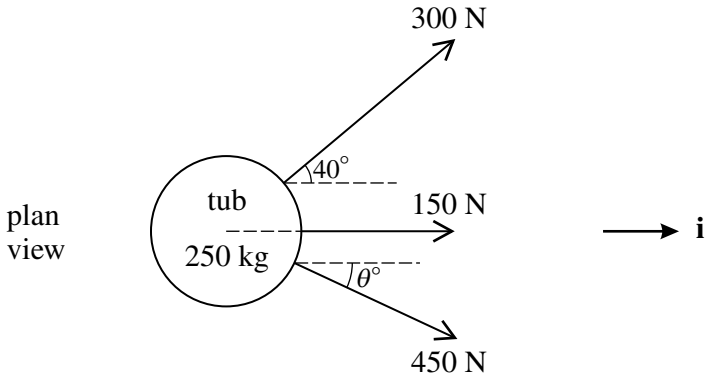


Fig. 8.2

Angle θ is acute and chosen so that the resultant of the three forces is in the **i** direction.

(iii) Determine the value of θ and the resultant of the three forces. **[6]**

With this resultant force, the tub moves with constant acceleration and travels 1 metre from rest in 2 seconds.

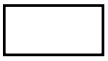
(iv) Show that the magnitude of the friction acting on the tub is 661 N, correct to 3 significant figures. **[5]**

When the speed of the tub is $1.8\,\text{m}\,\text{s}^{-1}$, it comes to a part of the floor where the friction on the tub is 200 N greater. The pulling forces stay the same.

(v) Find the velocity of the tub when it has moved a further 1.65 m. **[5]**

4 (i)

box P



box Q

4 (ii)

4 (iii)

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GCE

Mathematics (MEI)

Advanced Subsidiary GCE **4761**

Mechanics 1

Mark Scheme for June 2010

Q 1		mark	notes
(i)	$v^2 = 0^2 + 2\times 9.8\times 0.75$	M1	Use of $v^2 = u^2 + 2as$ with $u = 0$ and $a = \pm g$. Accept muddled units and sign errors.
		A1	Allow wrong or wrongly converted units not sign errors
	$v = \pm 3.8340...$ so 3.83 m s^{-1} (3. s. f.)	A1	cao [SC2 for 38.3… seen WWW and SC3 for 3.83… seen WWW]
		3	
		3	

Q 2		mark	notes
(i)	Resolving	M1	Resolving in at least 1 of horiz or vert. Accept $\sin \leftrightarrow \cos$. No extra terms.
	$\leftarrow\ 250\sin 70 = 234.92...$ so 235 N (3 s. f.)	A1	Either both expressions correct (neglect direction) or one correct in correct direction
	$\uparrow\ 250\cos 70 = 85.5050...$ so 85.5 N (3 s. f.)	A1 3	cao Both evaluated and directions correct
(ii)	$250 \div 2 = 125$ N	B1 1	Accept $125g$ only if tension taken to be $250g$ in (i)
		4	

Q 3		mark	notes
(i)	$\begin{pmatrix}-1\\14\\-8\end{pmatrix} + \begin{pmatrix}3\\-9\\10\end{pmatrix} + \mathbf{F} = 4\begin{pmatrix}-1\\2\\4\end{pmatrix}$	M1	N2L. Allow sign errors in applying N2L. Do not condone $\mathbf{F} = m\mathbf{a}$. Allow one given force omitted.
		M1	Attempt to add $\begin{pmatrix}-1\\14\\-8\end{pmatrix}$ and $\begin{pmatrix}3\\-9\\10\end{pmatrix}$
	$\mathbf{F} = \begin{pmatrix}-6\\3\\14\end{pmatrix}$	A1 A1 4	Two components correct cao
(ii)	$\mathbf{v} = \begin{pmatrix}-3\\3\\6\end{pmatrix} + 3\begin{pmatrix}-1\\2\\4\end{pmatrix} = \begin{pmatrix}-6\\9\\18\end{pmatrix}$ so $\begin{pmatrix}-6\\9\\18\end{pmatrix}$ m s^{-1}.	M1	$\mathbf{v} = \mathbf{u} + t\mathbf{a}$ with given $\mathbf{u}$ and $\mathbf{a}$. Could go via $\mathbf{s}$. If integration used, require arbitrary constant (need not be evaluated)
		A1	cao isw
	speed is $\sqrt{(-6)^2 + 9^2 + 18^2} = 21$ m s^{-1}.	M1	Allow -6^2 even if interpreted as – 36. Only FT **their v**.
		F1 4	FT their $\mathbf{v}$ only. [Award M1 F1 for 21 seen WWW]
		8	

Q 4		mark	notes
(i)	Diagram for P or Q Other diagram	B1 B1 2	Must be properly labelled with arrows Must be properly labelled with arrows consistent with 1st diagram Accept single diagram if clear.
(ii)	Let tension in rope be T N and accn ↑ a m s^{-2} For box P: N2L ↑ $1030 - 75g - T = 75a$ For box Q: N2L ↑ $T - 25g = 25a$	M1 A1 A1 3	N2L applied correctly to either part. Allow $F = mga$ and sign errors. Do not condone missing or extra forces. Direction of a consistent with equation for P. [Condone taking + ve downwards in either equation. +ve direction must be consistent in both equations to receive both A1s]
(iii)	tension is 257.5 N	M1 A1 2	Solving for T **their** simultaneous equations with 2 variables. cao CWO
		7	

Q 5		mark	notes
(i)	$270 - \arctan\left(\frac{6}{4}\right)$ = 213.69… so 214°	M1 A1 2	Award for arctan p seen where $p = \pm\frac{6}{4}$ or $\frac{4}{6}$, or equivalent cao
(ii)	Need $(-4 + 3k)\mathbf{i} + (-6 - 2k)\mathbf{j} = \lambda(7\mathbf{i} - 9\mathbf{j})$ * **either** so $\frac{-4+3k}{-6-2k} = \frac{7}{-9}$. or equivalent $k = 6$ **or** $-4 + 3k = 7\lambda$ $-6 - 2k = -9\lambda$ $k = 6$ **trial and error method**	M1 M1 A1 A1 M1 A1 A1 4	Attempt to get LHS in the direction of $(7\mathbf{i} - 9\mathbf{j})$. Could be done by finding (tangents of) angles. Accept the use of $\lambda = 1$. Attempt to solve **their** *. Allow = $\frac{7}{9}, \frac{9}{7}, -\frac{9}{7}$ Expression correct Award full marks for $k = 6$ found WWW Attempt to solve **their** *. Must have both equations. Correct equations Award full marks for $k = 6$ found WWW M1 any attempt to find the value of k and 'test' M1 Systematic attempt in (the equivalent of) **their** * Award full marks for $k = 6$ found WWW
		6	

Q6		mark	notes
(i)	Vertically		
	$y = 8t - 4.9t^2$	M1	Use of $s = ut + 0.5at^2$ with $g = \pm 9.8,\ \pm 10$. Accept $u = 0$ or 14.4… or $14.4\sin\theta$ or $u\sin\theta$ but not 12. Allow use of $+ 3.6$.
		A1	Accept derivation of – 4.9 not clear. cao.
	Horizontally $x = 12t$	B1	
		3	
(ii)	**either**		
	Require $y = -3.6$	M1	Equating **their** y to ±3.6 or equiv. Any form.
	so $-3.6 = 8t - 4.9t^2$		
	Use of formula or $4.9(t-2)(t+\frac{18}{49}) = 0$	M1	A method for solving a 3 term quadratic to give at least 1 root. Allow **their** y and re-arrangement errors.
	Roots are 2 and $-\frac{18}{49}$ $(= -0.367346\ldots)$	A1	WWW. Accept no reference to 2nd root [Award SC3 for $t = 2$ seen WWW]
	Horizontal distance is $12 \times 2 = 24$	M1	FT **their** x and t.
	so 24 m	F1	FT only **their** t (as long as it is +ve and is not obtained with sign error(s) e.g. –ve sign just dropped)
	or		
	Require $y = -3.6$	M1	Equating **their** y to ±3.6 or equiv. Any form.
	so $-3.6 = 8t - 4.9t^2$		
	Eliminate t between		
	$x = 12t$ and $-3.6 = 8t - 4.9t^2$	M1	Expressions in any form. Elimination must be complete
	so $0 = 3.6 + \frac{8x}{12} - \frac{4.9x^2}{144}$	A1	Accept in any form. May be implied.
	Use of formula or factorise	M1	A method for solving a 3 term quadratic to give at least 1 root. Allow **their** y and re-arrangement errors.
	+ve root is 24 so 24m	F1	FT from **their** quadratic after re-arrangement. Must be +ve.
	or		
	Methods that divide the motion into sections		
	Projection to highest point (A)		
	Highest point to level of jetty (B)		
	Level of jetty to sea (C)		
	Combination of A, B and C may be used	M1	Attempt to find times or distances for sections that give the total horizontal distance travelled
		M1	Correct method for one section to find time or distance
	(A) 0.8163.. s; 9.7959.. m: (B) 0.816…s; 9.7959.. m (C): 0.3673… s; 4.4081… m	A1	Any time or distance for a section correct
		A1	2nd time or distance correct (The two sections must not be A and B)
		A1	cao
		5	
		8	

Q 7		mark	notes
(i)			
(A)	4 m	B1	
(B)	$12 - (-4) = 16$ m	M1	Looking for distance. Need evidence of taking account of +ve and –ve displacements.
		A1	
(C)	$1 < t < 3.5$	B1	The values 1 and 3.5
		B1	Strict inequality
(D)	$t = 1$, $t = 3.5$	B1	Do not award if extra values given.
		6	
(ii)	$v = -8t + 8$	M1	Differentiating
		A1	
	$a = -8$	F1	
		3	
(iii)	$-8t + 8 = 4$ so $t = 0.5$ so 0.5 s	B1	FT **their** v.
	$-8t + 8 = -4$ so $t = 1.5$ so 1.5 s	B1	FT **their** v.
		2	
(iv)	**method 1**		
	Need velocity at $t = 3$		
	$v(3) = -8 \times 3 + 8 = -16$	B1	FT **their** v from (ii)
	either		
	$v = \int 32 \, dt = 32t + C$	M1	Accept $32t + C$ or $32t$. SC1 if $\int_3^4 32 dt$ attempted.
	$v = -16$ when $t = 3$ gives $v = 32t - 112$	A1	Use of **their** -16 from an attempt at v when t=3
	$y = \int (32t - 112) dt = 16t^2 - 112t + D$	M1	FT **their** v of the form $pt + q$ with $p \neq 0$ and $q \neq 0$.
			Accept if at least 1 term correct. Accept no D.
	$y = 0$ when $t = 3$		
	gives $y = 16t^2 - 112t + 192$	A1	cao.
	or		
	$y = -16 \times (t-3) + \frac{1}{2} \times 32 \times (t-3)^2$	M1	Use of $s = ut + \frac{1}{2}at^2$
		A1	Use of **their** -16 (not 0) from an attempt at v when t=3 and 32. Condone use of just t
		M1	Use of $t \pm 3$
		A1	cao
	(so $y = 16t^2 - 112t + 192$)		
	method 2		
	Since accn is constant, the displacement y is a quadratic function. Since we have $y = 0$ at $t = 3$ and $t = 4$		
	$y = k(t-3)(t-4)$	M1	Use of a quadratic function (condone no k)
		A1	Correct use of roots
		B1	k present
	When $t = 3.5$, $y = -4$		
	so $-4 = k \times \frac{1}{2} \times -\frac{1}{2}$	M1	Or consider velocity at $t = 3$
	so $k = 16$ (and $y = 16t^2 - 112t + 192$)	A1	cao. Accept k without y simplified.
		5	
		16	

Q8		mark	notes
(i)	N2L **i** direction $150 = 250a$ $a = 0.6$ so 0.6 m s^{-2}	 M1 A1 2	 Use of N2L. Allow $F = mga$. Accept no reference to direction
(ii)	150 N $-$ **i** direction	B1 B1 2	 Allow correct description or arrow [Accept '– 150 in **i** direction' for B1 B1]
(iii)	For force only in direction perp to **i** $300\sin 40 = 450\sin\theta$ $\theta = 25.37300\ldots$ so 25.4° (3 s. f.) In **i** direction $300\cos 40 + 150 + 450\cos\theta$ 786.4017… so 786**i** N (3 s. f.)	 M1 B1 A1 M1 A1 A1 6	 Resolution of both terms attempted. Allow $\sin \leftrightarrow \cos$ if in both terms. Allow 250 or 250g present. 300sin40 or 450sinθ Accept ± . Accept answer rounding to 25.5. Allow SC1 if seen in this part. Proper resolution attempted of 450 **and** 300. Allow $\sin \leftrightarrow \cos$ if in both terms Accept use of **their** θ or just θ. Either resolution correct. Accept **their** θ or just θ. Accept sin/cos consistent with use for cpt perpendicular to **i.** Accept no reference to direction cao. Allow SC1 WW
(iv)	Using $s = ut + 0.5at^2$ $1 = 0.5a \times 2^2$ $a = 0.5$ Using N2L in **i** direction 786.4017… $-F = 250 \times 0.5$ 661.4017… so 661 N (3 s. f.)	 M1 A1 M1 A1 E1 5	 Appropriate (sequence of) *suvat* [WW M0 A0] Use of $F = ma$ with **their** 786.4 and **their** a. No extra forces. Allow sign errors. All correct using **their** 786.4 and a Use of N2L clearly shown. (Accept 0.5 used WW)
(v)	Using N2L in **i** direction **either** $125 - 200 = 250a_1$ **or** (starting again) $786.4017\ldots - (200 + 661.4017\ldots) = 250\ a_1$ so $a_1 = -0.3$ Using $v^2 = u^2 + 2\ a_1 s$ $v^2 = 1.8^2 + 2 \times (-0.3) \times 1.65$ $v = 1.5$ so 1.5 m s^{-1}	 M1 F1 M1 F1 A1 5	 Use of $F = ma$ with **their** values. Allow 1 force missing FT only **their** 786… and **their** 661 Appropriate (sequence of) *suvat* with $u \neq 0$. Must be 'new' a obtained by using N2L. Only FT use of ± **their** a_1 cao
		20	